Defining Incumbent Protection Zones on the Fly: Dynamic Boundaries for Spectrum Sharing

Sudeep Bhattacharjia*, Abid Ullah*, Jung-Min “Jerry” Park*, Jeffery H. Reed*, David Gurney†, Bo Gao‡

*Department of Electrical and Computer Engineering, Virginia Tech, Blacksburg VA, USA
email: {sbhattar, abid, jungmin, reedjh}@vt.edu
†Motorola Solutions
email: David.P.Gurney@motorolasolutions.com
‡Institute of Computing Technology, Chinese Academy of Sciences, Beijing, China
email: gaobo@ict.ac.cn

Abstract—In spectrum sharing, a spatial separation region is defined around primary users (PUs) to protect them from secondary user (SU)-induced interference. This protection region—referred to by a number of names, such as an exclusion zone (EZ) or a protection zone (PZ)—has a static boundary, and this boundary is determined very conservatively to provide an additional margin of protection for the PUs. This legacy notion of interference protection is overly rigid, and often results in poor spectrum utilization efficiency. In this paper, we propose a novel framework for prescribing interference protection for the PUs that addresses some of the limitations of legacy EZs. Specifically, we introduce the concept of Multi-tiered Incumbent Protection Zones (MIPZ), and show that it can be used to dynamically adjust the PU’s protection boundary based on the radio environment, network conditions, and the PU interference protection requirement. MIPZ can serve as an analytical framework for quantitatively analyzing a given PZ to gain insights on and determine the tradeoffs between interference protection and spectrum utilization efficiency. It allows a number of SUs, say N, to operate closer to the PU, and improves the overall spectrum utilization efficiency while ensuring a probabilistic guarantee of interference protection to the PU. We leverage the combined power of database-driven spectrum sharing and stochastic optimization theory for dynamically computing the zone boundary and the value of N. Using extensive simulation results, we demonstrate that the proposed framework improves spectrum utilization efficiency by adapting to the changing interference environment through dynamic adjustments of the zone boundary.

Index Terms—Exclusion Zone, Protection Zone, Spectrum Sharing, Dynamic Spectrum Access, Spectrum Access System.

I. INTRODUCTION

In spectrum sharing, two types of stakeholders share the spectrum: incumbent users (a.k.a. Primary Users (PUs)) and Secondary Users (SUs). PUs have priority-access rights to their spectrum whereas SUs are allowed to opportunistically access the spectrum provided that the SU-induced interference is below a predefined threshold. To ensure interference protection, a static spatial separation region is defined around the PU where no co-channel and/or adjacent-channel transmission is allowed. This protected region is often called an Exclusion Zone (EZ). The EZ is the primary ex-ante (i.e., preventive) spectrum enforcement method that the Federal Communications Commission (FCC) and the National Telecommunications and Information Administration (NTIA) employ to protect non-federal and federal government PUs.

Defining the EZ boundary, inside which a PU enjoys exclusive access rights to the spectrum, is considered to be a challenging problem in spectrum sharing. The difficulty of the problem arises because of the following two conflicting requirements. On one hand, the area defined by the EZ must be sufficiently large to protect the PU from SU-induced interference. On the other hand, the EZ should not unnecessarily limit SUs spectrum access opportunities [1], otherwise the economic viability of spectrum sharing itself is undermined. Furthermore, when computing the EZ boundary, the effect of irregular terrain must also be considered in the path loss computations [2], which significantly increases the complexity of the already difficult problem.

Most of the existing methods, such as F-curves [3], tend to overly emphasize the first requirement, i.e., interference protection to the PUs [4], [5]. A good example of this can be seen in the TV bands. For example, to account for possible deep fades, the IEEE 802.22 working group specifications require detectors to have a sensitivity of \(-116\) dBm which corresponds to a safety margin of roughly \(20\) dB (equivalent to an increase in the radius of the EZ by \(110\) km) [6], [7]. However, in most situations, detectors do not face such severe fading, and hence the SUs are unnecessarily prohibited from using the band in question even though the probability of causing harmful interference to the PUs is extremely small. Such overly conservative designs of EZs significantly reduce the economic benefits of spectrum sharing [8] and, in some cases, may hinder its adoption due to lack of interest from the wireless industry stakeholders.

With the realization that opening up more spectrum to commercial applications has tremendous potential to spur economic growth and technological innovations [8], regulatory bodies, the wireless industry, and other stakeholders have taken active steps to address the technical as well as the policy challenges for realizing spectrum sharing. Among these challenges, two issues are especially critical in Federal-commercial spectrum sharing: (i) spectrum (rule) enforcement [9], [10], [11] and (ii) security and privacy [12], [13]. Spectrum enforcement involves employing technical and policy solutions for protecting incumbents from interference induced by lower-
Our framework, MIPZ, enables a seamless integration of two spectrum sharing approaches: database-driven and spectrum sensing-driven spectrum sharing.

B. AWS-3 Band

In AWS-3 band, the NTIA recently defined Coordination Zones (CZs) for sharing these bands with Wireless Broadband Systems (WBSs) [11]. The CZs are based on interference between satellite earth stations and WBSs. A CZ is not an EZ where SUs are not allowed to operate, but it is the area beyond which the earth station will not get interference from WBSs. In AWS-3 band, the NTIA recently defined Coordination Zones (CZs) for sharing these bands with Wireless Broadband Systems (WBSs) [11]. The CZs are based on interference between satellite earth stations and WBSs. A CZ is not an EZ where SUs are not allowed to operate, but it is the area beyond which the earth station will not get interference from WBSs [11]. WBSs have unencumbered access to the co-channel outside the CZ, but the ones that are willing to operate inside the CZ must trigger coordination with the federal incumbent.
Coordination process is initiated by WBS by submitting the detailed technical operating parameters to the federal point-of-contact who will respond to the request after assessing the possible interference caused at the incumbent. The WBS may or may not operate inside the CZ based on the response. Even if a WBS is allowed to operate inside the CZ, it must tolerate the possible harmful interference from the incumbents.

CZs are computed based on several factors, such as transmit power of both SU and PU, antenna gains in the direction of interference, time variations of antenna gains in the case of earth station operating with non-geostationary satellite systems, receiver susceptibility to interference, propagation effects of radio waves, mobility of earth station, etc. If the WBS can be shielded from the interference generated by the satellite earth station, then the size of the CZ is based on interference mitigation techniques at the WBS (e.g., using directional antennas to avoid interference to the incumbent).

III. PROPOSED FRAMEWORK: MULTI-TIERED INCUMBENT PROTECTION ZONES (MIPZ)

One of the main problems with conventional EZs is that they are overly large. The EZ boundary is defined conservatively so that the PUs are protected from interference even in the worst-case scenario. PUs experience higher interference when there is an interferer operating in a line-of-sight (LoS) region, and such interference is difficult to predict when the channel has small-scale fading characteristics.

The conservative approach for defining the conventional EZ boundary is also backed up by the following fact. Outside the EZ, the existing spectrum sharing model does not specify the limit on the number of simultaneous co-channel secondary transmissions — i.e., any SU can transmit in the co-channel as long as it is outside the PU’s EZ, and can co-exist with other SUs in the same band. Thus, the interference power received at the PU is not just the interference caused by a single SU, but in fact, it is the aggregate interference caused by multiple (theoretically infinite) SUs. In the absence of a spectrum access controller, it is quite understandable that regulators have to conservatively set the EZ boundary so that the PUs are protected from the worst-case aggregate interference.

The FCC in its Notice for Proposed Rule Making (NPRM) [15] acknowledges that the size of the EZ could be significantly reduced if there were a mechanism to control the number of SU transmissions outside the EZ. Regulators have stepped towards this direction by introducing database-driven spectrum sharing models where GDB, such as SAS in the 3.5 GHz band, acts as a spectrum controller. In a GDB-driven spectrum sharing, a SU queries the database, and accesses the channel only if the database responds with a spectrum access grant. Motivated by this, we propose MIPZ framework for GDB-driven spectrum sharing. MIPZ allows the spectrum controller to adjust the size of the PZ dynamically based on instantaneous interference conditions, and hence, allows SUs to exploit more spectrum opportunities than the legacy EZs.

First, we describe our framework assuming that the PU has a co-located transmitter (Tx) and receiver (Rx). Examples of co-located PUs are satellite earth stations, radar systems, etc. Then, in Section V, we provide some high-level insights on how to adapt our model for the non-colocated PUs. Our framework consists of the following three access zones.

A. No Access Zone (NAZ)

NAZ is the spatial separation region defined in the immediate vicinity of the PU where access to the spectrum is allowed only to the licensed incumbents. In our model, NAZ is a circle centered at the PU-Rx, and its boundary is computed dynamically based on the instantaneous radio and network conditions. The lower bound on the NAZ boundary is computed by considering interference in both directions: from SU to PU and from PU to SU.

B. Limited Access Zone (LAZ)

LAZ is a disk shaped annular region that lies just outside the NAZ. It shares its inner boundary with NAZ and the outer boundary with Unlimited Access Zone (which will be discussed shortly). Unlike the area outside conventional EZ, LAZ is the region which allows a limited number of co-channel SUs, say $N$, to transmit simultaneously. The upper bound on $N$ is carefully computed, which we shall discuss in detail in Section V. For a SU querying from LAZ region, the response of the spectrum database depends on the instantaneous number of other co-channel transmissions in the region. If the cardinality of other co-channel SUs operating in the LAZ is less than the upper bound of $N$, the querying SU is allowed to transmit in the co-channel, otherwise not. The outer boundary of the LAZ is computed using a propagation model such that the transmissions outside the LAZ cause negligible interference to the PU because of large path loss, hence, their contribution to aggregate interference can be ignored.

C. Unlimited Access Zone (UAZ)

UAZ is the region that lies outside the outer boundary of LAZ. In spirit, this region is similar to the area outside the conventional EZs where any number of SUs can transmit in the co-channel. Therefore, SUs have unencumbered access to the co-channel in the UAZ. The SU co-existence issues in the UAZ region is out of the scope of this paper.

Figure 1(a) shows our MIPZ framework. The outer rectangle denotes the analysis area that encompasses a co-located PU at the center. The NAZ is shown as a solid black area inside the inner boundary, where the black color signifies the absence of white space (spectrum opportunities) in that region. The gray disk between the inner and outer boundaries is the LAZ region, where the gray color signifies that only a limited number of white spaces are available. Outside the LAZ is the UAZ shown in white color, where the white color signifies that this region is affluent in white spaces.

D. Practical Considerations

In practice, the zone boundaries will not always be perfect circles as shown in Figure 1(a). Terrain variations, environmental effects, antenna radiation pattern, etc. cause the signal to attenuate differently in different directions resulting in
irregular zone boundaries. To consider the irregularity of the zone boundaries, we realize a sectorized model as shown in Figure 1(b). This model strikes an appropriate compromise between modeling realistic interference conditions and limiting modeling complexity. Here, each annular sector is a part of LAZ while the black irregular shape represents the NAZ. The inner boundary, as well as the upper bound on the number of SUs, needs to be defined for each LAZ sector.

Recall that no SU is allowed to operate inside the NAZ and SUs in UAZ do not contribute to the aggregate interference. Thus, in order to ensure that the aggregate interference power received by the PU is below its interference tolerance threshold \( I_{th} \), a centralized server/controller, such as the SAS, should govern the SU operations in the LAZ. The SAS should continuously monitor the instantaneous aggregate interference, and allow a new transmission inside LAZ only if the aggregate interference is lower than \( I_{th} \). However, the computational complexity of accurately monitoring the instantaneous aggregate interference in real-time is very challenging [20], and this makes such an approach impractical for applications such as the SAS.

To address the complexity of monitoring the instantaneous aggregate interference in real-time, we relax the system requirement by making the following assumption. Let us assume that the SUs can operate without significant performance degradation if they are ensured a probabilistic guarantee of interference protection. In other words, a PU may achieve its desired quality of service (QoS) if the aggregate interference \( I_{agg} \) from SUs is below \( I_{th} \) for \((1 - \epsilon)\) fraction of the time, where \( \epsilon \) is the probability that \( I_{agg} > I_{th} \).

\[
P(I_{agg} \leq I_{th}) \geq 1 - \epsilon
\]

Because of unpredictable nature of signal propagation, the notion of probabilistic guarantee is quite common in wireless applications. For example, the coverage regions of TV stations are based on F-curves which provide probabilistic guarantees that the signal reception is above a threshold.

**IV. AGGREGATE INTERFERENCE CHARACTERIZATION**

In this section, we first derive a closed-form expression for the probability distribution of co-channel interference caused at the PU by a single SU operating in a LAZ sector. This expression is valid for a SU transmitting in any LAZ sector provided that the relevant propagation parameters are available for that particular sector. Finally, in section IV-C, an expression for the probability distribution of aggregate interference is derived.

**A. Interference from a Single SU**

Let us consider a single SU operating inside a LAZ sector. We assume that SUs are uniformly distributed in space, therefore, the location of a SU is a two-dimensional uniform random variable. At first, this assumption might seem unreasonable as several studies have shown that mobile users tend to be clustered due to geographical factors, social gatherings, etc [21]. However, with multiple LAZ sectors, our framework can approximate the non-uniform SU distribution even if we consider uniform SU distribution in each sector. This can be achieved by considering different SU density in each sector.

In order to compute the path loss between SU and PU, let us consider a simplified propagation model with exponential path loss and shadowing. We choose this path loss model for the following two reasons: i) it is a popular path loss model for modeling large-scale outdoor channels, and has also been extensively used in prior 3GPP standards bodies [22], and ii) it facilitates us in deriving a closed-form analytical expression for the aggregate interference.

Beyond a reference distance \( d_0 \), the dB path loss \( P_L \) in the channel that links the SU and the PU is,

\[
P_L = a + b \log_{10} d + \psi,
\]

where \( a = P_L(d_0) - b \log_{10} d_0, P_L(d_0) \) is the path loss at the reference distance in dB, \( b = 10 \gamma \), \( \gamma \) is the path loss exponent, \( d \) is the distance between a SU and the PU in meters, and \( \psi \) is the log-normal (normal in dB scale) shadowing coefficient with zero mean and variance = \( \sigma^2 \). From here onwards, we shall consider all computations in dB unless explicitly stated otherwise; therefore, whenever we say normal distribution, it is actually a normal distribution in the log scale.

Suppose that SUs are uniformly distributed in an annular sector with the PU at the center. Then the distance between a SU and the PU can be represented with a random variable \( D \) whose probability density function (pdf) is given by equation (3) [23].

\[
f_D(d) = \frac{2d}{R_2^2 - R_1^2}, \quad R_1 \leq d \leq R_2.
\]

Here, \( R_1 \) and \( R_2 \) represent the radii of the inner and outer concentric circles, respectively, which combinedly define the annular LAZ sector.

Now, let us calculate the pdf of the second term of equation (2). This is basically a transformation of the random variable \( D \) to \( Y \), \( y = b \log_{10} d = g(d) \). We proceed as,

\[
f_Y(y) = f_D(g^{-1}(y)) \left| \frac{\partial y^{-1}(y)}{\partial y} \right| = \frac{2 \ln (10) 10^{y/b}}{b(R_2^2 - R_1^2)}, \quad b \log_{10} R_1 \leq y \leq b \log_{10} R_2.
\]
Since we consider normal shadowing, the pdf of third term of equation (2) is,
\[ f_\psi(\psi) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{\psi^2}{2\sigma^2}}. \]

Now that we know the pdf of the second and the third terms of equation (2), the resulting pdf of \( Z = Y + \psi \) is given by the following convolution integral [24],
\[
Z = \int_{-\infty}^{\infty} f_\psi(\psi) f_Y(z - \psi) d\psi
\]
\[
= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{\psi^2}{2\sigma^2}} 2 \ln(10) e^{2(z - \psi)/b} (R_2 - R_1) d\psi,
\]
where \( A_1 = z - b \log_{10} R_2 \) and \( B_1 = z - b \log_{10} R_1 \).

Let \( K_1 = \sqrt{\frac{2}{\pi b^2} e^{\frac{2 \ln(10) b^2}{b^2}}} \) and proceed.
\[
f_Z(z) = K_1 \int_{-\infty}^{\infty} e^{-\frac{\psi^2}{2\sigma^2}} e^{\frac{2(z - \psi) \ln 10}{b}} d\psi
\]
\[
= K_1 e^{\frac{z b^2}{2\sigma^2} b^2} B_2 \int_{A_2} e^{-\frac{k^2 \ln 10}{2\sigma^2}} dk, \tag{4}\]
where \( k = \psi + \frac{2 \ln 10}{b} \), \( A_2 = z - b \log_{10} R_2 + \frac{2 \ln 10}{b} \) and \( B_2 = z - b \log_{10} R_1 + \frac{2 \ln 10}{b} \).

Letting \( p = \frac{k}{\sqrt{\sigma}} \) and \( K_2 = K_1 \sqrt{\frac{2}{\pi b^2}} e^{\frac{2 \ln(10) b^2}{b^2}} \), equation (4) becomes,
\[
f_Z(z) = \frac{2}{\sqrt{\pi}} K_2 e^{\frac{2 \ln 10}{b}} \int_{A_3} e^{-r^2} dr
\]
\[
= K_2 e^{\frac{2 \ln 10}{b}} \{ erf(B_3) - erf(A_3) \},
\]
where \( erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt \),
\[
A_3 = \frac{1}{\sqrt{2\sigma}} \left( z - b \log_{10} R_2 + \frac{2 \ln 10}{b} \right), \quad \text{and}
\]
\[
B_3 = \frac{1}{\sqrt{2\sigma}} \left( z - b \log_{10} R_1 + \frac{2 \ln 10}{b} \right).
\]

Finally, the pdf of \( P_L \) in equation (2) is,
\[
f_{P_L}(p_L) = K_2 e^{\frac{2(p_L - a) \ln 10}{b}} \{ erf(B_4) - erf(A_4) \}, \tag{5}\]
where \( A_4 = \frac{1}{\sqrt{2\sigma}} \left( p_L - a - b \log_{10} R_2 + \frac{2 \ln 10}{b} \right) \), and \( B_4 = \frac{1}{\sqrt{2\sigma}} \left( p_L - a - b \log_{10} R_1 + \frac{2 \ln 10}{b} \right) \).

Let \( P_t \) denote the transmit power of SU in dBm. Then, the interference power received by the PU receiver is,
\[ I_{SU} = P_t - P_L. \tag{6} \]

Using equations (5) and (6), the pdf of \( I_{SU} \) is,
\[
f_{I_{SU}}(i_{su}) = K_2 e^{\frac{2(p_L - a) ln 10}{b}} \{ erf(B_3) - erf(A_3) \}, \tag{7}\]
where \( A_3 = \frac{1}{\sqrt{2\sigma}} \left( P_t - i_{su} - a - b \log_{10} R_2 + \frac{2 \ln 10}{b} \right) \)
and \( B_3 = \frac{1}{\sqrt{2\sigma}} \left( P_t - i_{su} - a - b \log_{10} R_1 + \frac{2 \ln 10}{b} \right) \).

As mentioned before, Equation (7) is valid for any SU operating in any LAZ sector. When specific values of \( a, b, P_t, \sigma, R_1 \) and \( R_2 \) pertaining to \( i_{th} \) SU operating in \( j^{th} \) LAZ sector are plugged into Equation (7), the pdf of \( I_{SU_{j,i}} \) is obtained. Here, \( I_{SU_{j,i}} \) denotes the pdf of interference power at the PU receiver due to the transmission from \( j^{th} \) SU operating in a randomly chosen location inside the \( j^{th} \) LAZ sector.

B. Approximating \( I_{SU} \) Distribution as a Normal

The pdf in equation (7) looks notoriously complex as its kernel cannot be recognized as that of any of the standard pdfs. This poses as a major road-block in our quest of finding the closed form expression for aggregate interference, \( I_{agg} \). Let us rewrite equation (7) as follows,
\[
f_{I_{SU}}(i_{su}) = \frac{K_3}{\omega^2 - 4} g_1(i_{su}) g_2(i_{su}), \tag{8}\]
where \( g_2(\omega) = erf(g_3(\omega)) - erf \left( \frac{g_3(\omega) - b \log_{10} \omega}{\sqrt{2\sigma}} \right) \).
\( \omega = R_2/R_1 \), \( g_3(i_{su}) \) and \( g_1(i_{su}) \) are linear and exponential functions of \( i_{su} \) respectively. \( K_3 \) is a non-negative constant.

From the definition of the \( erf \) function, the plot of \( g_2(\omega) \) can be approximated as a Gaussian pdf. This approximation is fairly accurate when \( b \log_{10} \omega \) is small. Since \( b = 10^\gamma \) (\( \gamma \) is the path loss exponent) and \( \sigma \) (standard deviation of shadow fading) are channel characteristics, a wireless system designer has no control over them. Therefore, the accuracy of the Gaussian approximation for the plot of \( g_2(\omega) \) depends on \( \omega \), where \( \omega > 1 \) (since \( R_2 > R_1 \)). The approximation is highly accurate when \( \omega \) is small. As \( \omega \) becomes larger, the bell shaped curve of \( g_2(\omega) \) starts deviate from the Gaussian pdf. Figure 2(a) shows the comparative plots of \( g_2(i_{su}) \) against the closest normal pdf for different values of \( \omega \).

The function \( g_1(i_{su}) \) has the kernel of an exponential distribution. Using the fact that the product of an exponential kernel and a Gaussian kernel results in another Gaussian kernel, \( f_{I_{SU}}(i_{su}) \) is a Gaussian pdf. Note that this approximation is accurate only when \( \omega \) is small. Large value of \( \omega \) causes \( g_2(i_{su}) \) and hence \( f_{I_{SU}}(i_{su}) \) to deviate from the normal pdf resulting in a non-zero approximation error (\( \Delta f_{SU} \)). We define \( \Delta f_{SU} \) as the Euclidean norm of the difference between actual and approximated distributions of \( I_{SU} \).

In Figure 2(b), the actual plots of \( f_{I_{SU}}(i_{su}) \) from Equation (7) and its complementary cumulative distribution function (ccdf) are compared against the pdf and the ccdf of normal approximation respectively. For generating these plots, typical practical values were plugged in for all other variables (\( a = 37 \) dB, \( b = 20 \), \( \sigma = 3 \), \( P_t = 23 \) dBm, \( R_2 = 126 \) km). Then, the parameters of the normal approximation are obtained by fitting a least squares normal curve to the samples of \( f_{I_{SU}}(i_{su}) \). We
can observe a close similarity between the two pdfs, especially when $\omega$ is small. The plot of $\Delta I_{SU}$ in Figure 2(c) shows that $\Delta I_{SU}$ increases with increase in $\omega$. As expected, the plot also shows that the approximation error is a function of $\omega$ but not of the actual values of $R_1$ and $R_2$. Another important observation is that for any $\omega$, $\Delta I_{SU}$ increases as the ratio $\gamma/\sigma$ increases.

In our framework, $\omega$ is not significantly large. As we shall discuss in Section V, typical value of $R_2$ is 126 km and $R_1$ ranges from 50 km to 126 km, which implies $\omega$ ranges from 1 to 2.52. At this value, $\Delta I_{SU}$ is fairly negligible. Therefore, we conclude that $I_{SU}$ can be approximated as a normal distribution. This allows us to obtain the closed-form expression for the pdf of $I_{SU}$. Later, we shall further justify this approximation by showing that it has negligible effect on the overall performance of PU and SU networks.

**C. Aggregate Interference**

The next step is to find the distribution of $I_{agg}$, which is the summation of random variables, $I_{SU_{i,j}}$.

$$I_{agg} = \sum_{j=1}^{T} \sum_{i=1}^{N_j} I_{SU_{i,j}}. \quad (9)$$

Here, $T$ denotes the total number of LAZ sectors and $N_j$ is the total number of SUs operating in the $j^{th}$ LAZ sector. Note that equation (9) is valid only in standard units (Watts or milliWatts), but not in dB units. Since the distribution of $I_{SU_{i,j}}$ (in standard units) is log-normal, $I_{agg}$ has the distribution of summation of log-normal random variables.

It has been shown that the summation of log-normal random variables can be approximated by another log-normal [25]. Several approximation techniques have been proposed [25], [26]. The most widely used approximations are the ones proposed by Fenton-Wilkinson [27], Schwartz-Yeh [28] and Mehta et al. [29]. Fenton-Wilkinson is a simple and computationally efficient algorithm for approximating the mean and variance of the resulting log-normal distribution. While it provides a very good approximation in the tail region of the cdf curve, Fenton-Wilkinson is usually bad in the body region. Schwartz-Yeh provides a good approximation in the body region at the cost of added computational complexity, but unlike Fenton-Wilkinson, it doesn’t do well in the tail region. Mehta et al. provide a flexible mechanism that allows a user to choose the focus of the approximation. However, its computational complexity increases exponentially with the increase in the number of random variables being summed, which makes it the least favorable for using in real-time high traffic applications like the SAS.

In order to provide probabilistic guarantee of interference protection to the PU, the following inequality must be satisfied.

$$P(I_{agg} \leq I_{th}) \geq 1 - \epsilon$$

i.e.,

$$P\left(\sum_{j=1}^{T} \sum_{i=1}^{N_j} I_{SU_{i,j}} \leq I_{th}\right) \geq 1 - \epsilon. \quad (10)$$

From inequality (10), it is clear that we are interested in the tail portion of the complementary cdf of $I_{agg}$. Fenton-Wilkinson fits our purpose because it provides a log-normal approximation that is most accurate in the tail region [30]. Moreover, it performs well even with the summation of non-identically distributed log-normal variables (summands). This is desired in our case because the distribution of $I_{SU}$ might be different for different LAZ sectors when sectors have different sets of parameters such as $\gamma$, $P_{ts}$ and $\sigma$. Furthermore, Fenton-Wilkinson provides a closed-form solution for the mean and variance of the resulting log-normal distribution, making it easier to implement in the SAS. The closed-form solutions are given in equations (11) and (12) [27].

$$\sigma_{agg}^2 = \ln\left(\frac{\sum_{j=1}^{T} \sum_{i=1}^{N_j} (e^{2\mu_{i,j}+\sigma_{i,j}^2} - 1)}{\sum_{j=1}^{T} \sum_{i=1}^{N_j} (e^{\mu_{i,j}+\sigma_{i,j}^2/2})} + 1\right) \quad (11)$$

$$\mu_{agg} = \ln\left(\sum_{j=1}^{T} \sum_{i=1}^{N_j} (e^{\mu_{i,j}+\sigma_{i,j}^2/2})\right) - \frac{\sigma_{agg}^2}{2}, \quad (12)$$

where $\mu_{i,j}$ and $\sigma_{i,j}^2$ denote the mean and variance of individual summand. Similarly, $\mu_{agg}$ and $\sigma_{agg}^2$ are mean and variance of the resulting log-normal distribution: $I_{agg}$ in our case.

The above equations are valid for natural logarithm, and they must be scaled appropriately when working with other logarithms ($\log_{10}$ in our case).
V. DETERMINING THE MIPZ BOUNDARIES

Our framework defines NAZ, LAZ and UAZ regions based on two boundaries: outer boundary and inner boundary (Figure 1(a)). The details of the boundary definitions are described in the next two sub-sections. In the following discussions, it is assumed that the PU’s Tx and Rx are colocated. The non-colocated scenario will be discussed briefly at the end of the section.

A. Static Outer Boundary

The spectrum sharing etiquette in the UAZ region is exactly the same as that in the region outside conventional EZ. The SAS provides unencumbered access to the co-channel in the UAZ which forces us to define the outer boundary conservatively, just like the conventional EZ boundary. Otherwise, the PU may not be guaranteed an adequate interference protection either due to LoS interference from peak points in some terrain areas, or due to the aggregate interference from SUs. On the other hand, there are some possible spectrum opportunities near the conventional EZ boundary which are unnecessarily thwarted because of conservative boundary definition. To exploit such opportunities, we leverage the conventional EZ boundary definition as a starting point and use it as the outer boundary of our framework, and then explore spectrum opportunities inside it. This also allows us to make a direct comparison between the conventional EZ and our framework in terms of spectrum utilization.

We define the outer boundary of our framework in the same way as the regulators define the conventional EZ boundary, i.e., based on the maximum distance at which the PU can get interference from the SUs. The maximum distance depends on several factors such as SU transmit power, type of modulation and coding, PU Rx antenna gain, PU’s interference protection and QoS requirement, etc. The Longley-Rice propagation model in point-to-point is used for pathloss calculations in determining the outer boundary. Furthermore, the outer boundary is static because it is computed based on the worst-case interference conditions rather than the instantaneous radio conditions. As a specific example, a CZ of radius 126 km is defined for a satellite Earth Station in AWS-3 band, located in Patuxent River, Maryland, USA.

B. Dynamic Inner Boundary

The inner boundary separates the NAZ and LAZ regions. It is clear from Section III that the SAS allows only a limited number of SUs to operate in the LAZ region. Usually, the wireless network conditions are dynamic. For example, at peak times of the day, more SUs want to access the channel, while in the maintenance hours, only a few of them do so. To cope with the changing network conditions, it is desired that the size of the LAZ be dynamic so that the spectrum resources can be allocated on the fly. Note that the size of the LAZ plays a major role in determining the number of available spectrum resources in the region. In the previous subsection, we discussed that the outer boundary is static. Therefore, we define the inner boundary based on instantaneous network conditions, and make the LAZ region dynamic in size.

First, let us define the upper and lower bounds on \( R_1 \), the inner boundary. Clearly, the upper bound on \( R_1 \) is the outer boundary \( R_2 \). When \( R_1 = R_2 \), our model becomes equivalent to the conventional EZ. When \( R_1 < R_2 \), there is a non-zero area available in the LAZ region. This is where the SAS allows a limited number of SUs, say \( N \), to operate. Small \( R_1 \) implies large area in the LAZ region, and apparently, it seems that this translates to a higher value of \( N \). However, small \( R_1 \) has two major implications. The first issue with small \( R_1 \) is that it results in large \( \Delta I_{SU} \). Figure 2(b) shows that our approximation predicts lower probability of interference in the tail region as compared to that given by the exact closed-form expression of \( I_{SU} \). As \( R_1 \) gets small, the difference increases. The implication is that when \( R_1 \) is small and our approximation is used to compute the available number of spectrum resources in the LAZ, it computes \( N \) that is larger than the actual \( N \) permitted in the LAZ. This endangers the PU’s interference protection, and therefore, forces us to define a lower bound on \( R_1 \), say \( R_{lb1}^{(1)}, R_{lb2}^{(1)} \) is computed based on the maximum tolerable \( \Delta I_{SU} \).

Another issue with small \( R_1 \) is that it brings the LAZ region closer to the PU. Referring to Figure 2(b), small \( R_1 \) causes the \( I_{SU} \) ccdf to shift to the right, and increases the probability that \( I_{SU} > I_{th} \). This forces us to define another lower bound on \( R_1 \), say \( R_{lb1}^{(2)}, R_{lb2}^{(2)} \), based on the interference protection requirement of the PU. \( R_{lb1}^{(2)} \) is the distance at which a single SU endangers the protection requirement of the PU, as this forms a sort of lower bound. It is calculated using \( I_{th}, \epsilon \) and pathloss equations.

When PU-Tx and PU-Rx are colocated, \( R_{lb1}^{(2)} \) depends on the interference from SU for a desired interference protection requirement of the PU. We define the interference tolerance level of PU in terms of outage probability, which is the probability that the received signal power coming from a co-channel SU is greater than a predefined interference threshold. The outage probability at the PU due to interference from a co-channel SU located at \( R_{lb1}^{(2)} \) in a shadow fading channel with variance \( \sigma^2 \) is calculated as follows,

\[
\epsilon = P (I_{SU} \geq I_{th}) = Q \left( \frac{I_{th} - \bar{I}_{SU}}{\sigma} \right)
\]

(13)

where \( Q(.) \) is the Gaussian Q function, and \( \bar{I}_{SU} \) is the mean interference power which is given by,

\[
\bar{I}_{SU} = P_{ts} - a - 10\gamma \log_{10} R_{lb1}^{(2)},
\]

(14)

where \( a = 10\gamma\log_{10} \left( \frac{4\pi f}{c} \right) \), \( f \) is the radio frequency and \( c \) is the speed of propagation of the radio wave through the medium. Plugging (14) in (13) and rearranging gives \( R_{lb1}^{(2)} \).

\[
R_{lb1}^{(2)} = 10 \left( \frac{e^{-\frac{1}{2} \left( I_{th} - P_{ts} + a - \bar{I}_{SU} \right)}}{\epsilon} \right).
\]

(15)

The co-located PUs, such as radars and satellite earth stations, have significantly higher transmit power (upto 90 dBm) compared to the SUs (20 – 33 dBm for the small cell LTE base stations) [31]. When there is a large power discrepancy between the PU and SU, the interference from the PU to SU is a concern. To address this, we introduce
a third lower bound on \( R_1 \), say \( R^{(3)}_1 \), is the minimum distance from the PU at which a SU can achieve its desired QoS level. If the QoS of the SU is also defined in terms of probabilistic guarantee of interference protection, \( R^{(3)}_1 \) is given by equation (15) when \( I_{th} \) and \( \epsilon \) are replaced with the interference threshold and outage probability of the SU, and \( P_s \) is replaced with the transmit power of the PU.

The smallest \( R_1 \) that satisfies all three bounds is \( R_m \).

\[
R_m = \max \left( R^{(1)}_{1h}, R^{(2)}_{1h}, R^{(3)}_{1h} \right).
\]

Apart from many advantages of GDB-driven spectrum sharing, it is often argued that the database might, at times, contain the stale information. While sensing-driven spectrum sharing provides real-time spectrum availability information, the cost of cooperation among the sensing nodes is extremely high. Recently, studies have shown that a fusion of GDB-driven and sensing-driven spectrum sharing can provide a better spectrum sharing experience [32], [33]. We allow our framework to enable the marriage of database-driven and sensing-driven spectrum sharing approaches by adding a value \( \alpha \) to \( R_m \), where \( \alpha \), which can be negative or positive, is determined by the sensing results. Adding \( \alpha \) to \( R_m \) allows the database to refine the inner zone boundary based on real-time sensing results. Moreover, incorporating sensing results enhances the performance of MIPZ framework in finding the spectrum opportunities that are left uncaptured by the simplified propagation model used in the analytical analysis. The details pertaining to the computation of \( \alpha \) is, however, out of the scope of this paper. We shall pursue the detailed study of the tuning parameter, \( \alpha \), and the problem of combining the database contents with the sensing results in our future work.

\[
R_{min} = \alpha + \max (R^{(1)}_{1h}, R^{(2)}_{1h}, R^{(3)}_{1h})\). \quad (16)
\]

On the other hand, when \( R_2 \) is large, the LAZ region lies far from the PU-Rx. The ccdf curve of Figure 2(b) shifts to the left. From this, we expect to achieve large \( N \). However, large \( R_1 \) means small area for spectrum sharing in the LAZ region, and to address the co-existence issues among SUs, \( N \) should be small. These conflicting requirements make the problem of defining the inner boundary challenging.

Let \( \lambda \) denote the total number of spectrum requests coming from uniformly distributed SUs in an area between \( R_{min} \) and \( R_2 \) of a LAZ sector. Then, the total number of spectrum requests in an annular region between \( R_1 \) and \( R_2 \), \( \lambda_{LAZ} \), is,

\[
\lambda_{LAZ} = \frac{\lambda (R_2^2 - R_1^2)}{(R_2^2 - R_{min}^2)}. \quad (17)
\]

When multiple SUs co-exist in a band, the co-existence among the SUs is also an issue. Suppose that a maximum of \( \rho \) SUs can co-exist in the area between \( R_{min} \) and \( R_2 \). From here onwards, we use SU to refer to a SU cell with a Tx at the center and a single Rx at the cell edge. \( \rho \) is computed by using SU’s coverage area, its transmit power, required Signal-to-Noise-and-Interference-Ratio (SINR) at the SU-Rx, antenna parameters, path loss exponent and shadow fading environment. For simplicity, let us assume that co-existence is a function of the total area available for SUs and the area of each SU cell, i.e., \( \rho = \frac{(R_2^2 - r_{su}^2)}{r_{su}^2} \), where \( r_{su} \) is the cell radius of the SU. Then, the total number of SUs that can co-exist in an area between \( R_1 \) and \( R_2 \), \( \rho_{LAZ} \), is,

\[
\rho_{LAZ} = \frac{(R_2^2 - R_1^2)}{r_{su}^2}. \quad (18)
\]

Ideally, the desired number of SUs in the LAZ region is the minimum of \( \lambda_{LAZ} \) and \( \rho_{LAZ} \). Assuming \( R_1 \) and \( R_2 \) are already defined, there is no incentive in allowing more than \( \lambda_{LAZ} \) SUs because only \( \lambda_{LAZ} \) SUs are requesting access to the co-channel. Also, allowing more than \( \rho_{LAZ} \) SUs causes unnecessary co-existence interference among the SUs.

Based on the above discussions, we formulate the following stochastic optimization problem for finding optimum \( R_1 \) that maximizes \( N \) while minimizing \( \omega \), and also satisfies the PU’s protection criteria. Recall that minimizing \( \omega \) ensures that the approximation error, \( \Delta I_{SU} \), is minimized. In this formulation, it is assumed that there is a single LAZ sector and all SUs operating in the LAZ have same transmission parameters, resulting in same distribution of \( I_{SU} \) for all SUs.

Maximize : \( N - \omega \)

subject to : \( P \left( \sum_{i=1}^{N} I_{SU_i} \leq I_{th} \right) \geq 1 - \epsilon \) \quad (19)

\[
R_{min} \leq R_1 \leq R_2 \leq R_{2min} \leq R_2 \leq R_{2min} \leq R_{2min} \leq R_2 
\]

Now, let us extend the above problem formulation to the case when there are \( T \) LAZ sectors. Suppose \( N^{(j)}, R_{min}^{(j)}, R_1^{(j)}, R_2^{(j)}, \lambda^{(j)}_{LAZ}, \rho^{(j)}_{LAZ} \) and \( I_{SU_j} \) denote the number of SUs, \( R_{min} \), \( R_1 \), \( R_2 \), \( \lambda_{LAZ} \), \( \rho_{LAZ} \) and \( I_{SU} \) of \( j^{th} \) sector respectively. Then, the optimization problem (19) can be reformulated as (20).

Maximize : \( \sum_{j=1}^{T} \left( \eta^{(j)} N^{(j)} - \omega \right) \)

subject to : \( P \left( \sum_{j=1}^{T} \sum_{i=1}^{N^{(j)}} I_{SU_{ij}} \leq I_{th} \right) \geq 1 - \epsilon \)

\[
R_{min}^{(j)} \leq R_1^{(j)} \leq R_2^{(j)} \leq R_{2min}^{(j)} \leq R_2^{(j)} \leq R_{2min}^{(j)} \leq R_{2min}^{(j)} \leq R_2^{(j)} 
\]

When all SUs within a LAZ sector have the same link capacity (Mbps/Hz), the weights \( \eta^{(j)} \) correspond to the relative spectral capacities (or relative spectral efficiencies) of SUs in different LAZ sectors. A higher number of SUs is desired in the sector that has higher link capacity for each SU. Link capacities can be different when different types of SUs (e.g., LTE, WiFi, etc.) or SUs with different operating parameters (e.g., \( P_{ts}, r_{su} \), etc.) operate in different LAZ sectors. Terrain characteristics, which might be different in different LAZ sectors, affect the propagation characteristics, \( \gamma \) and \( \sigma \), which in turn affect the link capacities of SUs. However, if all LAZ sectors have SUs with the same link capacity, then \( \eta^{(j)} = 1 \) for all \( j \), and the objective function in (20) simplifies to a regular sum of \( (N^{(j)} - \omega) \).
Optimization problems (19) and (20) are mixed-integer non-linear programming problems because they require $N_j, j = 1, \ldots, T$ to be integers, and the interference constraint is non-linear. Several algorithms such as cutting-plane [34] and branch-and-bound [35] can be used to solve this kind of problems. But often, due to their computational complexity, Genetic Algorithm (GA) is preferred. A GA is a heuristic search algorithm for solutions of optimization problems that starts from a random initial guess and attempts to find the best solution under some criteria [36]. Problems (19) and (20) can be easily solved using GAs.

In practice, both $\lambda$ and $\rho$ vary with time. The query requests arriving at the SAS is high during peak hours, while it is quite low during maintenance hours. Similarly, $\rho$ changes when the SUs change their coverage area, transmit power, etc. Changes in these parameters and other operating parameters of the PU and the SUs also changes the distribution of $I_{SU_j}$ and hence $I_{agg}$. Assuming that all these parameters are available to the SAS beforehand, it solves the optimization problem (20) whenever it expects these parameters to change. The SAS then responds to the spectrum queries coming from the SUs based on the solution of problem (20) — i.e., it allows a maximum of $N$ spectrum access grants inside the LAZ at any given time.

Until now, we assumed that the PU has colocated Tx and Rx; while in practice, PUs may have non-colocated Tx and Rx. Examples of non-colocated PUs are TV stations and any other broadcast systems. Our derivations can be easily extended to a non-colocated PU by adding a margin, $\Delta I$, to the $I_{th}$ of the PU, where $\Delta I$ is a function of the path loss between PU-Tx and PU-Rx located at the edge of the coverage area.

VI. SIMULATION RESULTS

In this section, we present simulation results for demonstrating the performance of our proposed framework. In the first half of this section, we compare the results from our analytical solution with those from Monte-Carlo simulations, and justify that the normal approximation for characterizing the pdf of $I_{SU}$ has negligible impact on the PU’s interference protection. Then, in the later half, we present results to show that our framework dynamically adjusts the size of LAZ, computes the allowed number of SUs in the LAZ based on dynamic network conditions, and maximizes the overall spectrum utilization.

Let us define the database coverage area as a 300 km by 300 km square with a co-located PU at the center. The

![Fig. 3: cdf of aggregate interference experienced by PU](image)

<table>
<thead>
<tr>
<th>TABLE I: Sample parameters for simulations</th>
</tr>
</thead>
<tbody>
<tr>
<td>Parameter</td>
</tr>
<tr>
<td>-----------------------------------------</td>
</tr>
<tr>
<td>Radio frequency, $f$</td>
</tr>
<tr>
<td>Radiation pattern</td>
</tr>
<tr>
<td>SU transmit power, $P_{ts}$</td>
</tr>
<tr>
<td>SU cell size, $r_{su}$</td>
</tr>
<tr>
<td>Total spectrum requests from SUs, $\lambda$</td>
</tr>
<tr>
<td>Channel bandwidth ($W_s$)</td>
</tr>
<tr>
<td>Path loss exponent, $\gamma$</td>
</tr>
<tr>
<td>Standard deviation of shadow fading, $\sigma$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>TABLE II: Four scenarios considered in Figure 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Scenario</td>
</tr>
<tr>
<td>-----------------</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>2</td>
</tr>
<tr>
<td>3</td>
</tr>
<tr>
<td>4</td>
</tr>
</tbody>
</table>

PU considered for this simulation study is an actual MetSat earth station in the AWS-3 band located at Petuxant River, Maryland, USA. For this PU, regulators have defined a circular EZ of radius 126 km, and therefore we set $R_2 = 126$ km. The area outside $R_2$ is the UAZ region, and we divide it into square grids of side 2$r_{su}$ km, each of which hosts a SU cell with radius $r_{su}$. We do not have access to the specific operating parameters of the PU, so, let us assume its transmit power, $I_{th}$ and $\epsilon$ as 60 dBm, −100 dBm and 0.1 respectively unless otherwise explicitly stated. For the LTE based SUs, measurements have shown that there is no influence on the SU throughput when the radar interference power is below −50 dBm [37]. Therefore, we assume that for proper operation of SUs, the interference from the incumbent should be below −50 dBm at least 0.9 fraction of the time. For simplicity, let us assume that the LAZ consists of a single sector, and has the same propagation environment ($\gamma$ and $\sigma$) as the UAZ region. Also, the SUs in UAZ and LAZ have identical transmission parameters as outlined in Table I. Furthermore, we assume that sensing results are not available to the database; therefore $\alpha$ in equation (16) is set to zero. Using these parameters, the SAS solves the optimization problem (19), and computes the optimum values of $R_1$ and $N$. We use those results to study the performance of the primary and secondary networks in terms of interference protection and spectrum utilization respectively.

A. PU Interference Protection: Our Approximation versus Monte-Carlo Simulations

The closed-form expression for $I_{agg}$ was derived based on the following two approximations: i) pdf of $I_{SU}$ (in standard units) is log-normal, and ii) sum of log-normals is another log-normal. In order to justify that the PU’s interference protection is not compromised by making such approximations, we perform a Monte-Carlo (MC) based simulation study. First, the optimization problem in (19) is solved by making the aforementioned approximations to obtain $N$ and $R_1$. Using these results and equations (11) and (12), we obtain the cdf plot of $I_{agg}$.

For performing the MC simulations, $N$ SUs are uniformly distributed in the area between $R_1$ and $R_2$, and the aggregate
interference power received at the PU is calculated using equations (2), (6) and (9). Then, we perform 50,000 MC iterations, and compare the empirical ccdf of $I_{agg}$ against the ccdf obtained from closed-form expressions. Figure 3 shows a close similarity between the two plots for different scenarios outlined in Table II. This verifies that our approximation does not compromise the interference protection of the PU—the probabilistic guarantee of interference protection. $P(I_{agg} \leq -100 \text{ dBm}) \geq 0.9$, is always achieved. Our approximation slightly underestimates $I_{agg}$ for large $\gamma$ and small $\sigma$ values, and slightly overestimates it for small $\gamma$ and large $\sigma$ values.

B. Spectrum Utilization: Adapting to Dynamic Network Conditions

To study the effect of model parameters on the secondary spectrum utilization, we define spectrum utilization in terms of Area Sum Capacity (ASC). ASC is the sum of channel capacity values of each co-existing SU within the SAS coverage area. Throughout the simulations, we assume that SU is a cell of radius $r_{su}$, which consists of a single Tx at the center and a single Rx at the cell edge. The channel capacity ($C_{SU}$) of a SU operating in a channel of bandwidth $W_s$ is calculated using the Shannon capacity formula.

$$C_{SU} = W_s \log_2(1 + \text{SINR})$$  \hspace{1cm} (21)

Here, the SINR at the SU-Rx is given by,

$$\text{SINR} = \frac{P_{ts}/P_L(r_{su})}{n_s W_s + I_{P2S} + I_{S2S}}$$  \hspace{1cm} (22)

where, $P_L(r_{su})$ is the path loss between the SU-Tx and SU-Rx, $n_s$ is the thermal noise power at the SU-Rx, $I_{P2S}$ is the PU to SU interference and $I_{S2S}$ is the aggregate interference power at the SU from other co-existing SUs.

Now, if we assume all SUs use the same bandwidth, the SU ASC (units = bits per second) is computed as,

$$\text{ASC} = W_s \sum_{i=1}^{N_T} \log_2(1 + \text{SINR}_i)$$  \hspace{1cm} (23)

where, $N_T$ represents the total number of SUs in the system (both LAZ and UAZ), and $\text{SINR}_i$ denotes the SINR at the $i^{th}$ SU-Rx.

1) Effect of $I_{th}$: The effect of $I_{th}$ on $N$, $R_1$ and ASC is shown in Figure 4. As $I_{th}$ increases, the SAS extends LAZ towards the PU by making $R_1$ smaller until it becomes equal to $R_{min}$. Increased area in the LAZ and high $I_{th}$ implies that more SUs (increased $N$) can be accommodated in the LAZ. Although the increased number of SUs in the LAZ lowers the SINR of existing SUs in both UAZ and LAZ regions due to increased $I_{S2S}$ and decreases their capacity, Figure 4(c) shows that the ASC gain from added SUs is significant enough to overcome the loss. Next observation in Figure 4(b) is that around $I_{th} = -76$ dBm, $R_{min}$ kicks in and does not allow $R_1$ to decrease further even when $I_{th}$ increases. Also, since the upper bound of $N$ depends on $R_1$ (recall equations (17) and (18)), $N$ saturates and so does ASC. Another important observation in Figure 4(a) is the low sensitivity of $N$ on $\epsilon$. When $\epsilon$ is large, the SAS packs more SUs in the LAZ region by increasing the size of the LAZ. But when the size of LAZ increases (i.e., $R_1$ decreases), the distribution of $I_{SU}$ also changes. For small $R_1$, the ccdf of $I_{SU}$ moves towards higher values of $i_{su}$ (see Figure 2(b)) which increases the probability that a SU causes interference to the PU. Because this change in $I_{SU}$ applies to all SUs in the LAZ, $N$ cannot be increased.
by a huge factor without violating the PU interference criteria. Therefore, we do not see a significant increase in $N$ even when $\epsilon$ increases by an order of magnitude.

2) Effect of $\lambda$: The effect of $\lambda$ on $N$, $R_1$ and ASC is shown in Figure 5 for different $I_{th}$ values at $\epsilon = 0.1$. When there are less number of SU requests, the SAS maximizes $N$ by increasing the size of the LAZ, i.e., making $R_1$ smaller. Small $\lambda$ implies small $\lambda_{LAZ}$, therefore, the upper bound on $N$ is $\lambda_{LAZ}$ but not $\rho_{LAZ}$ (recall the last constraint of (19)). Consequently, increasing $\lambda_{LAZ}$ by decreasing $R_1$ maximizes $N$, and hence, the ASC. However, the lower bound on $R_1$ prevents the SAS from decreasing it beyond $R_{min}$ as noticed in Figure 5(b) for $I_{th} = -90$ dBm. Another observation from Figure 5(b) is that $R_{(2)}_{th}$ for sensitive PUs (having small $I_{th}$) is large, and this results in large $R_{min}$. Large $R_{min}$ decreases $\lambda_{LAZ}$ which ultimately results in smaller $N$, and hence, a smaller gain in ASC as compared to the less sensitive PUs.

3) Effect of $r_{su}$: Figure 6 shows that our framework adapts to the change in SU cell size, and addresses the co-existence among SUs in the LAZ. From equation (18), $\rho_{LAZ}$ decreases when $r_{su}$ increases for any $I_{th}$ value. When $\lambda$ and $I_{th}$ both are large, $\rho_{LAZ}$ dictates the upper bound on $N$. So, in order to maximize $N$, SAS increases the size of the LAZ by decreasing $R_1$. However, for sensitive PUs, LAZ cannot be increased by a huge factor, otherwise the PU interference criteria may not be satisfied. Therefore, $N$ is small when $I_{th}$ is small. As $N$ decreases with increasing $r_{su}$, a decrease in ASC gain is observed. Recall our assumption that the SU cell consists of a single Tx at the center and a single Rx at the cell edge. Large SU cell size implies reduced SINR at the SU-Rx, which causes the ASC gain to decline sharply even when $N$ does not.

4) Effect of $P_{ts}$: Our framework also adapts to the change in SU transmit power in the LAZ. The results are summarized in Figure 7. When $P_{ts}$ is large, the SAS reduces the size of LAZ by increasing $R_1$ to protect the PU from interference. Large $R_1$ implies small $\lambda_{LAZ}$ and $\rho_{LAZ}$, the upper bounds on $N$. As a result, $N$ is small. Nevertheless, this decrease in $N$ does not necessarily reduce the ASC. With high $P_{ts}$, SU Rxs in the LAZ experience increased SINR which results in a gain in ASC. This gain overcomes the loss in ASC due to decreased $N$, specially when $N$ is large, such as for $I_{th} = -95$ dBm and $P_{ts} = 16$ dBm in Figure 7(c). However, when $N$ is very small, such as for $I_{th} = -100$ dBm and $P_{ts} = 23$ dBm in Figure 7(c), the ASC loss due to decreased $N$ is higher than the gain achieved from increased SINR, and hence, the overall ASC gain from LAZ is small. This provides us a valuable insight that $P_{ts}$ can be optimized for maximizing the ASC.

C. Economic Merit of MIPZ

In Figure 8, we illustrate the possible economic merit of implementing our proposed framework. The outer boundary represents the current EZ defined by NTIA [11] for a AWS-
3 based MetSat Earth station, and the green annular region is the LAZ region defined by our model for a realistic set of parameters. The introduction of the LAZ region serves approximately 10 million people of Richmond, VA, Washington D.C. and Baltimore, MD, which would otherwise lie in the NTIA-defined EZ. With a bandwidth of 15 MHz, this area represents about 150 million MHz-POPs for a wireless operator. Using Verizon’s valuation of the nearby AWS band in their proposed spectrum swap, this is worth approximately $132 million per auction period [1]. [38].

Although we analyzed the economic merit of MIPZ for this particular incumbent, similar analysis can be done for other incumbents as well. The economic merit varies based on the location of the incumbent on the map.

VII. CONCLUSION

In this paper, we introduced the concept of multi-tiered dynamic PZs for ex-ante spectrum enforcement in GDB-driven spectrum sharing. The proposed framework allows a limited number of SUs to operate closer to the PU, and improves the overall spectrum utilization while ensuring a probabilistic guarantee of interference protection to the PUs. By making some reasonable assumptions, we derived a closed-form expression of the aggregate interference power received by the PU, and used it to dynamically adjust the size of the PZ boundary. Using extensive simulation results, we showed that our framework can effectively adapt to the changing interference environment to increase spectrum utilization efficiency.

VIII. ACKNOWLEDGEMENT

This work was partially sponsored by NSF through grants 1314598, 1265886 and 1431244; by Motorola Solutions; and our framework can effectively adapt to the changing interfer-

boundary. Using extensive simulation results, we showed that

By making some reasonable assumptions, we derived a closed-

probabilistic guarantee of interference protection to the PUs.

limited number of SUs to operate closer to the PU, and
dynamic PZs for ex-ante spectrum enforcement in GDB-
spectrum swap, this is worth approximately

Verizon’s valuation of the nearby AWS band in their proposed
customers as well. The economic merit varies based on the

termination of IEEE International Conference in Communications (ICC), 2015.

REFERENCES


